

VOLUME OF A CYLINDER

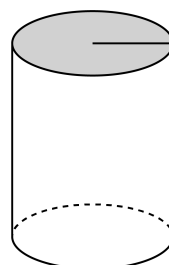
The volume of a cylinder is the area of its base multiplied by its height:

$$V = B \cdot h$$

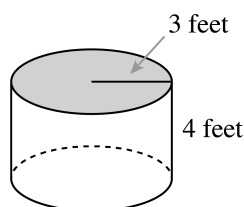
Since the base of a cylinder is a circle of area $A = r^2\pi$, we can write:

$$V = r^2\pi h$$

For additional information, see the Math Notes box in Lesson 10.1.2 of the *Core Connections, Course 3* text.



Example 1

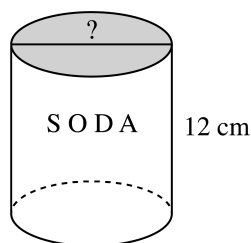


Find the volume of the cylinder above.

Use a calculator for the value of π .

$$\begin{aligned}\text{Volume} &= r^2\pi h \\ &= (3)^2\pi(4) \\ &= 36\pi \\ &\approx 113.10 \text{ ft}^3\end{aligned}$$

Example 2



The soda can above has a volume of 355 cm^3 and a height of 12 cm. What is its diameter?
Use a calculator for the value of π .

$$\begin{aligned}\text{Volume} &= r^2\pi h \\ 355 &= r^2\pi(12) \\ \frac{355}{12\pi} &= r^2 \\ 9.42 &\approx r^2 \\ r &\approx 3.07 \\ \text{diameter} &= 2r \approx 2(3.07) \approx 6.14 \text{ cm}\end{aligned}$$

Problems

Find the volume of each cylinder.

- | | | |
|--|---|---|
| 1. $r = 5 \text{ cm}$
$h = 10 \text{ cm}$ | 2. $r = 7.5 \text{ inches}$
$h = 8.1 \text{ inches}$ | 3. diameter = 10 cm
$h = 5 \text{ cm}$ |
| 4. base area = 50 cm^2
$h = 4 \text{ cm}$ | 5. $r = 17 \text{ cm}$
$h = 10 \text{ cm}$ | 6. $d = 29 \text{ cm}$
$h = 13 \text{ cm}$ |

Find the missing part of each cylinder.

7. If the volume is 5175 ft^3 and the height is 23 ft, find the diameter.
8. If the volume is $26,101.07 \text{ inches}^3$ and the radius is 17.23 inches, find the height.
9. If the circumference is 126 cm and the height is 15 cm, find the volume.

Answers

- | | | |
|--------------------------|----------------------------|-----------------------------|
| 1. 785.40 cm^3 | 2. 1431.39 in.^3 | 3. 392.70 cm^3 |
| 4. 200 cm^3 | 5. 9079.20 cm^3 | 6. 8586.76 cm^3 |
| 7. 16.93 ft | 8. 28 inches | 9. $18,950.58 \text{ cm}^3$ |

SURFACE AREA OF A CYLINDER

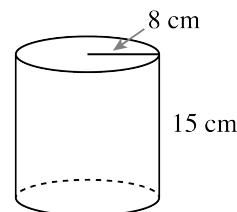
The surface area of a cylinder is the sum of the two base areas and the lateral surface area.
The formula for the surface area is:

$$SA = 2r^2\pi + \pi dh \quad \text{or} \quad SA = 2r^2\pi + 2\pi rh$$

where r = radius, d = diameter, and h = height of the cylinder. For additional information, see the Math Notes box in Lesson 10.1.3 of the *Core Connections, Course 3* text.

Example 1

Find the surface area of the cylinder at right.
Use a calculator for the value of π .



Step 1: Area of the two circular bases

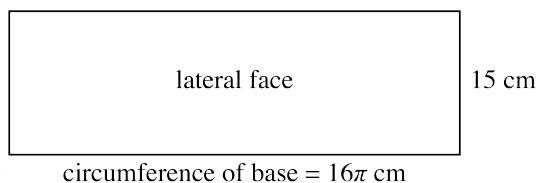
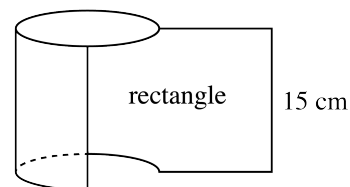
$$2[(8 \text{ cm})^2\pi] = 128\pi \text{ cm}^2$$

Step 2: Area of the lateral face

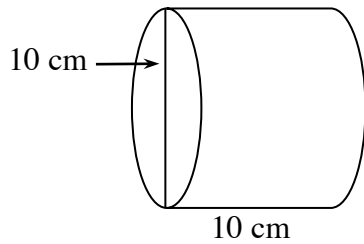
$$\pi(16)15 = 240\pi \text{ cm}^2$$

Step 3: Surface area of the cylinder

$$128\pi \text{ cm}^2 + 240\pi \text{ cm}^2 = 368\pi \text{ cm}^2 \approx 1156.11 \text{ cm}^2$$

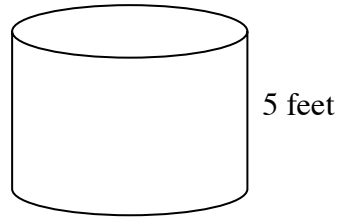


Example 2



$$\begin{aligned} SA &= 2r^2\pi + 2\pi rh \\ &= 2(5)^2\pi + 2\pi \cdot 5 \cdot 10 \\ &= 50\pi + 100\pi \\ &= 150\pi \approx 471.24 \text{ cm}^2 \end{aligned}$$

Example 3



If the volume of the tank above is $500\pi \text{ ft}^3$, what is the surface area?

$$\begin{aligned} V &= \pi r^2 h \\ 500\pi &= \pi r^2 (5) \\ \frac{500\pi}{5\pi} &= r^2 \\ 100 &= r^2 \\ 10 &= r \end{aligned} \qquad \begin{aligned} SA &= 2r^2\pi + 2\pi rh \\ &= 2 \cdot 10^2\pi + 2\pi(10)(5) \\ &= 200\pi + 100\pi \\ &= 300\pi \approx 942.48 \text{ ft}^2 \end{aligned}$$

Problems

Find the surface area of each cylinder.

- | | | |
|--|---|---|
| 1. $r = 6 \text{ cm}$, $h = 10 \text{ cm}$ | 2. $r = 3.5 \text{ in.}$, $h = 25 \text{ in.}$ | 3. $d = 9 \text{ in.}$, $h = 8.5 \text{ in.}$ |
| 4. $d = 15 \text{ cm}$, $h = 10 \text{ cm}$ | 5. base area = 25,
height = 8 | 6. volume = 1000 cm^3 ,
height = 25 cm |

Answers

- | | | |
|--------------------------|---------------------------|---------------------------|
| 1. 603.19 cm^2 | 2. 626.75 in.^2 | 3. 367.57 in.^2 |
| 4. 824.69 cm^2 | 5. 191.80 in.^2 | 6. 640.50 cm^2 |

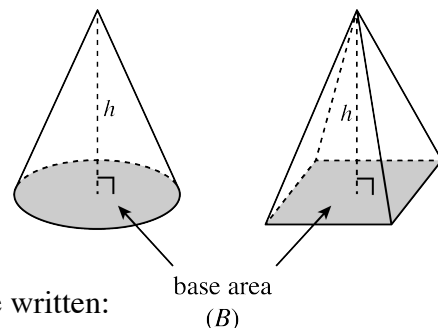
The volume of a pyramid is one-third the volume of the prism with the same base and height and the volume of a cone is one third the volume of the cylinder with the same base and height. The formula for the volume of the pyramid or cone with base B and height h is:

$$V = \frac{1}{3} Bh$$

For the cone, since the base is a circle the formula may also be written:

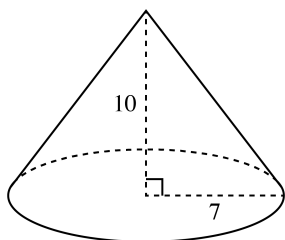
$$V = \frac{1}{3} r^2 \pi h$$

For additional information, see the Math Notes box in Lesson 10.1.4 of the *Core Connections, Course 3* text.



Example 1

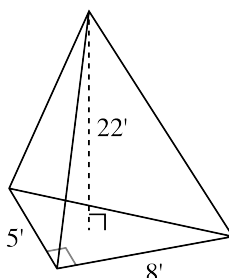
Find the volume of the cone below.



$$\begin{aligned} \text{Volume} &= \frac{1}{3} (7)^2 \pi \cdot 10 \\ &= \frac{490\pi}{3} \\ &\approx 513.13 \text{ units}^3 \end{aligned}$$

Example 2

Find the volume of the pyramid below.



$$\begin{aligned} \text{Base is a right triangle} \\ B &= \frac{1}{2} \cdot 5 \cdot 8 = 20 \\ \text{Volume} &= \frac{1}{3} \cdot 20 \cdot 22 \\ &\approx 146.67 \text{ ft}^3 \end{aligned}$$

Example 3

If the volume of a cone is 4325.87 cm^3 and its radius is 9 cm, find its height.

$$\begin{aligned} \text{Volume} &= \frac{1}{3} r^2 \pi h \\ 4325.87 &= \frac{1}{3} (9)^2 \pi h \\ 12977.61 &= 81\pi h \\ \frac{12977.61}{81\pi} &= h \\ 51 \text{ cm} &\approx h \end{aligned}$$

Problems

Find the volume of each cone.

- $r = 4 \text{ cm}$
 $h = 10 \text{ cm}$
- $r = 2.5 \text{ inches}$
 $h = 10.4 \text{ inches}$
- $d = 12 \text{ inches}$
 $h = 6 \text{ inches}$
- $d = 9 \text{ cm}$
 $h = 10 \text{ cm}$
- $r = 6\frac{1}{3} \text{ ft}$
 $h = 12\frac{1}{2} \text{ ft}$
- $r = 3\frac{1}{4} \text{ ft}$
 $h = 6 \text{ ft}$

Find the volume of each pyramid.

- | | | | | | |
|----|---|----|---|----|--|
| 7. | base is a square with
side 8 cm
$h = 12$ cm | 8. | base is a right triangle
with legs 4 ft and 6 ft
$h = 10\frac{1}{2}$ ft | 9. | base is a rectangle with
width 6 in., length 8 in.
$h = 5$ in. |
|----|---|----|---|----|--|

Find the missing part of each cone described below.

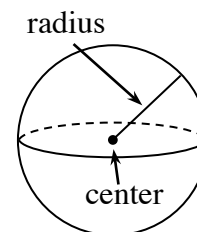
10. If $V = 1000 \text{ cm}^3$ and $r = 10$ cm, find h .
11. If $V = 2000 \text{ cm}^3$ and $h = 15$ cm, find r .
12. If the circumference of the base = 126 cm and $h = 10$ cm, find the volume.

Answers

- | | | | | | |
|-----|-----------------------|-----|-----------------------|-----|------------------------|
| 1. | 167.55 cm^3 | 2. | 68.07 in^3 | 3. | 226.19 in^3 |
| 4. | 212.06 cm^3 | 5. | 525.05 ft^3 | 6. | 66.37 ft^3 |
| 7. | 256 cm^3 | 8. | 42 ft^3 | 9. | 80 in^3 |
| 10. | 9.54 cm | 11. | 11.28 cm | 12. | 4211.24 cm^3 |

For a sphere with radius r , the volume is found using $V = \frac{4}{3} \pi r^3$.

For more information, see the Math Notes box in Lesson 10.1.5 of the *Core Connections, Course 3* text.



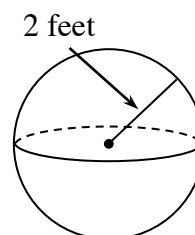
Example 1

Find the volume of the sphere at right.

$$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi \cdot 2^3 = \frac{32\pi}{3} \text{ ft}^3 \text{ (exact answer)}$$

or using $\pi \approx 3.14$,

$$\frac{32(3.14)}{3} \approx 33.49 \text{ ft}^3 \text{ (approximate answer)}$$



Example 2

A sphere has a volume of $972\pi \text{ un.}^3$. Find the radius.

Use the formula for volume and solve the equation for the radius.

$$V = \frac{4}{3} \pi r^3 = 972\pi$$

Substitution.

$$4\pi r^3 = 2916\pi$$

Multiply by 3 to remove the fraction.

$$r^3 = \frac{2916\pi}{4\pi} = 729$$

Divide by 4π to isolate r .

$$r = \sqrt[3]{729} = 9$$

To undo cubing, take the cube root.

Problems

Use the given information to find the exact and approximate volume of the sphere.

- | | | |
|-----------------------|--|---|
| 1. radius = 10 cm | 2. radius = 4 ft | 3. diameter = 10 cm |
| 4. diameter = 3 miles | 5. circumference of great circle = 12π un. | 6. circumference of great circle = 3π un. |

Use the given information to answer each question related to spheres.

- If the radius is 7 cm, find the volume.
- If the diameter is 10 inches, find the volume.
- If the volume of the sphere is $36\pi \text{ un.}^3$, find the radius.
- If the volume of the sphere is $\frac{256\pi}{3} \text{ un.}^3$, find the radius.

Answers

- | | |
|---|--|
| 1. $\frac{4000\pi}{3} \approx 4186.67 \text{ cm}^3$ | 2. $\frac{256\pi}{3} \approx 267.94 \text{ ft}^3$ |
| 3. $\frac{500\pi}{3} \approx 523.33 \text{ cm}^3$ | 4. $\frac{9\pi}{2} \approx 14.13 \text{ mi}^3$ |
| 5. $288\pi \approx 904.32 \text{ un.}^3$ | 6. $\frac{9\pi}{2} \approx 14.13 \text{ mi}^3$ |
| 7. $\frac{1372\pi}{3} \approx 1436.75 \text{ cm}^3$ | 8. $\frac{500\pi}{3} \approx 523.60 \text{ in.}^3$ |
| 9. $r = 3 \text{ units}$ | 10. $r = 4 \text{ units}$ |